Non-Smooth Secondary Source Distributions in Wave Field Synthesis

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Introduction

Wave Field Synthesis (WFS) is a well-established sound field synthesis (SFS) technique that uses a dense distribution of loudspeakers (*secondary sources*) arranged around an extended listening area. The physical foundations of WFS assume a smooth contour on which the secondary sources are located. Practical systems are often of rectangular shape, which constitutes a non-smooth secondary source contour. The resulting effects on the synthesized sound field are investigated in this paper. In order to isolate the artifacts of one edge from other aspects, semi-infinite rectangular arrays are considered. It is shown that edges can result in considerable amplitude and spectral deviations. These results are supplemented by a case-study where an existing array is investigated.

Wave Field Synthesis

The physical background of SFS is given by the Helmholtz integral equation (HIE) [1]. This fundamental acoustic principle states that the sound field in a region V is uniquely given by the pressure and its direction gradient on the region's boundary ∂V, that has to be smooth and simply connected. Furthermore the volume has to be free of sources and scattering objects. The straightforward application of the HIE to SFS would require the useage of two types of loudspeakers realizing ideal monopole and dipole secondary sources. Various solutions have been developed for monopole-only SFS, for instance the single layer potential or equivalent scattering approach [2]. WFS applies a stationary-phase approximation to the HIE to achieve monopole-only reproduction [3]. The applied approximations hold for large distances between the secondary sources and the listener and/or for high-frequencies. The synthesized sound field $P(\boldsymbol{x},\omega)$ reads in the temporal spectrum domain [4]

$$P(\boldsymbol{x},\omega) = \oint_{\partial V} \underbrace{-2\,a(\boldsymbol{x}_0)\,\frac{\partial S(\boldsymbol{x}_0,\omega)}{\partial n(\boldsymbol{x}_0)}}_{D(\boldsymbol{x}_0,\omega)}\, G(\boldsymbol{x}-\boldsymbol{x}_0,\omega)\,\mathrm{d}A(\boldsymbol{x}_0) \tag{1}$$

for $\boldsymbol{x} \in V$ and $\boldsymbol{x}_0 \in \partial V$ and inward pointing normal. The desired sound field (*primary/virtual source*) is denoted by $S(\boldsymbol{x}_0,\omega)$, $a(\boldsymbol{x}_0)$ denotes a window function for the selection of active secondary sources, $G(\boldsymbol{x}-\boldsymbol{x}_0,\omega)$ the Green's function and $D(\boldsymbol{x}_0,\omega)$ the secondary source driving function. For SFS, the Green's function is realized by loudspeakers placed on ∂V. For two-dimensional synthesis the Green's function constitutes a line source and for three-dimensional a point source. Practical setups consist often of a contour ∂V embedded in a plane, ideally leveled with the ears of the listener. Instead of line sources, point sources are used resulting in a dimensionality mismatch. Such configurations employ so called *2.5-dimensional synthesis*. In order to avoid the resulting artifacts, the effect of non-smooth secondary source contours is investigated for the two-dimensional case first. Due to the geometry of typical listening rooms, most loudspeaker arrays are of rectangular shape. Their edges violate the assumptions made on ∂V for the HIE. In order to isolate the effects of an edge, a stepwise transition from a linear secondary source contour with infinite length to a semi-infinite rectangular secondary source contour is performed in the next section.

Semi-Infinite Rectangular Secondary Source Distribution

The synthesized sound field for an infinitely long linear secondary source distribution located on the x-axis is given as [5]

$$P(\boldsymbol{x},\omega) = \int_{-\infty}^{\infty} D(x_0,\omega)\, G(\boldsymbol{x}-\boldsymbol{x}_0,\omega)\,\mathrm{d}x_0, \tag{2}$$

with $\boldsymbol{x} = (x,y)^T$ and $\boldsymbol{x}_0 = (x_0,0)^T$. In order to derive the sound field for a semi-infinite rectangular secondary source distribution two steps are performed: (i) truncation of the infinitely long secondary source distribution and (ii) superposition with a 90° rotated and truncated linear secondary source distribution. The first step is modeled by windowing the driving function with the heaviside step function $\epsilon(x_0)$ [6]

$$P(\boldsymbol{x},\omega) = \int_{-\infty}^{\infty} \epsilon(x_0)\, D(x_0,\omega)\, G(\boldsymbol{x}-\boldsymbol{x}_0,\omega)\,\mathrm{d}x_0. \tag{3}$$

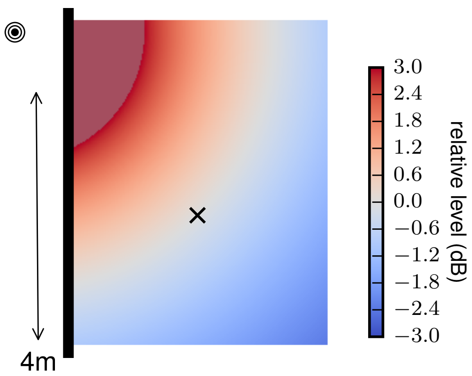
A spatial Fourier transformation with respect to x_0 results in

$$\tilde{P}_\epsilon(k_x,y,\omega) = \tilde{D}_\epsilon(k_x,\omega)\,\tilde{G}(k_x,y,\mathbf{0},\omega), \tag{4}$$

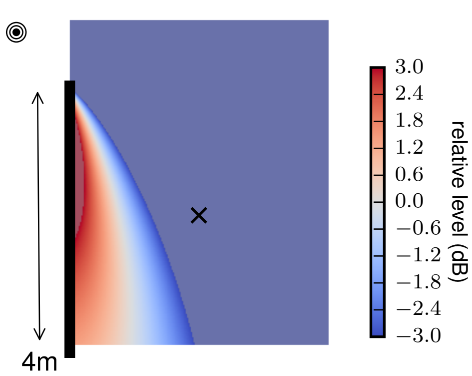
where k_x denotes the wavenumber and the subscript ϵ quantities for the semi-infinite case. The wavenumber-frequency spectrum $\tilde{D}_\epsilon(k_x,\omega)$ of the truncated driving function is given as

$$\tilde{D}_\epsilon(k_x,\omega) = \frac{1}{2\,\pi}\left(\pi\,\delta(k_x) + \frac{1}{\mathrm{j}\,k_x}\right) *_{k_x} \tilde{D}(k_x,\omega). \tag{5}$$

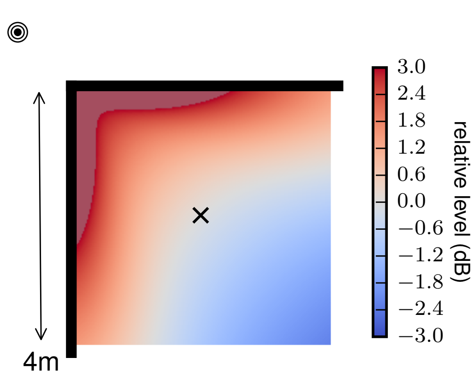
For the propagating part, the spectrum $\tilde{G}(k_x,y,\mathbf{0},\omega)$ of the Greens function is bandlimited to $|\frac{\omega}{c}| < k_x$. This



(a) Infinite



(b) Semi-infinite, case (i)



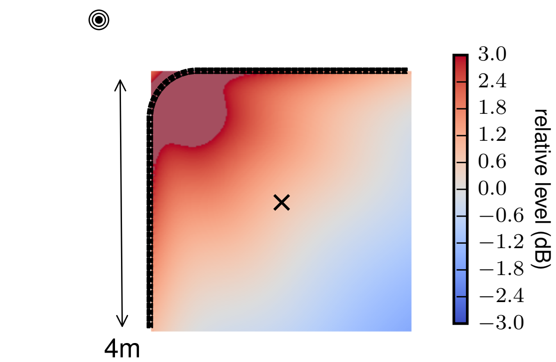
(c) Semi-infinite rectangular, case (ii)

Figure 1: 2D synthesis of a line source by secondary line sources using WFS. The edge is located at $(0, 0)$ m. The source is located at a distance of $R = 4$ m and an angle of $\alpha = 135°$ from the reference position indicated by $\times$. A monochromatic source with $f = 500$ Hz is simulated. The level is normalized to the reference position for the infinitely long secondary source distribution.

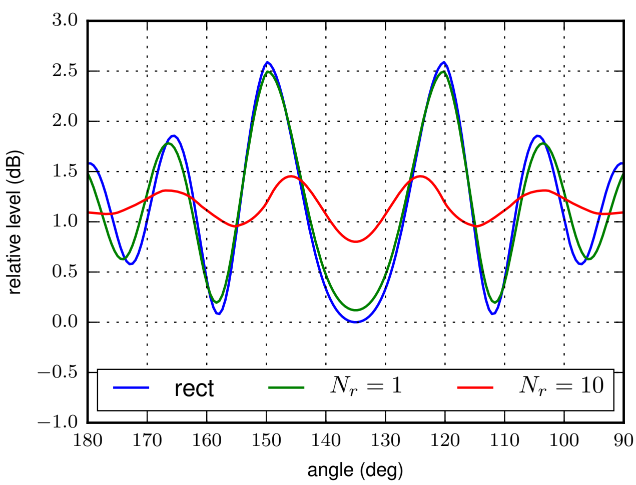
truncation results in Gibbs phenomena [6] being present in the synthesized sound field. The sound field synthesized by a semi-infinite rectangular secondary source distribution can be derived by superimposing two rotated semi-infinite linear secondary source distributions.

For the following examples, a semi-infinite rectangular secondary source contour along the positive x- and negative y-axis is considered. The edge is located at the origin. The rotation can be performed by replacing x with $-y$ in Equation (3). Figure 1 shows the relative level of the synthesized sound field for the three steps discussed above. The level has been normalized (0 dB) to the position indicated by the cross for the infinitely long linear secondary source distribution. The synthesis of a point source with $f = 500$ Hz at the position indicated by the circles (upper left) is shown. The two-dimensional driving function for a virtual line source given in [4] was used. No tapering window has been applied since this increases the artifacts when comparing different virtual source positions. This is due to the fact, that the tapering window is applied to the active secondary sources only. However, for the given geometry this would result in increased amplitude variations when only one of the secondary source distributions is active compared to the case when both are active. The tapering towards the edge in the first case would not be present in the second case.

Figure 1a for an infinitely long linear secondary source distribution shows the expected amplitude decay of a virtual line source. Truncating the secondary source distribution, as shown in Fig. 1b, results in considerable deviations. The situation improves for the semi-infinite rectangular secondary source distribution shown in Fig. 1c. However, severe deviations from the expected amplitude decay can still be observed. These additionally depend on the source position and frequency. For further evaluation, a simulation has been carried out were the level at the reference position for various virtual source positions is calculated. For a fixed distance of the virtual point source to this position, a constant level would be expected. The virtual point source was sequentially moved



(a) relative level of synthesized sound field for $N_r = 10$, $\alpha = 135°$



(b) relative level at reference position for various α

Figure 2: 2D synthesis of a line source by secondary line sources using a sampled semi-infinite rounded rectangular secondary source distribution. The source is located at a distance of $R = 4$ m and an angle α from the reference position indicated by $\times$. A monochromatic source with $f = 500$ Hz is simulated. The level is normalized to the reference position for $\alpha = 135°$. The number of secondary sources in the rounded edge is denoted by N_r.

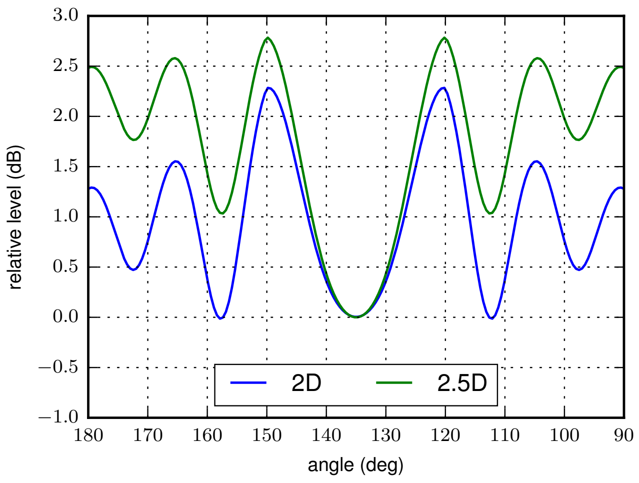


Figure 3: 2D/2.5D synthesis by secondary line/point sources using a sampled semi-infinite rectangular secondary source distribution. Relative level at reference position for synthesis of a line/point source. A monofrequent source with $f = 500$ Hz is simulated. The source is located at a distance of $R = 4$ m and angle α from the reference position. The level is normalized for $\alpha = 135°$.

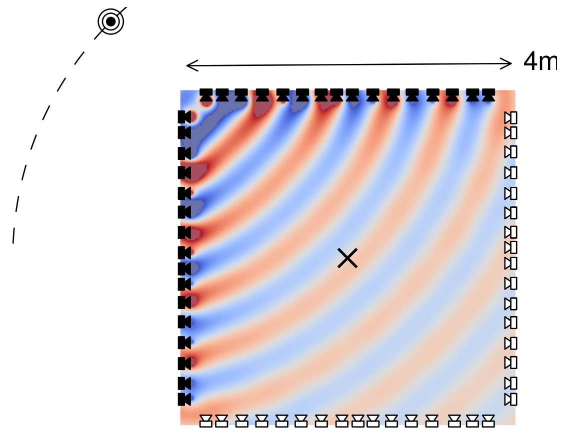


Figure 4: 2.5D synthesis of a monochromatic point source by WFS. The source is located at a distance of $R = 4$ m and angle $\alpha = 135°$ from the reference position indicated by $\times$. A monochromatic source with $f = 500$ Hz is simulated. The level is normalized to the reference position for $\alpha = 135°$.

from the center left position ($\alpha = 180°$) to the upper center ($\alpha = 90°$) around the reference position. The resulting level is shown in Figure 2b (rect). Amplitude variations up to 2.5 dB can be observed. These will be clearly audible for a listener at the reference position.
A potential countermeasure to these amplitude variations could be a smoothed secondary source distribution by replacing the edge with a quarter circle. Figure 2a shows the level of the synthesized sound field for $N_r = 10$ secondary sources in the rounded corner. The artifacts become less but still some deviations are present. The amplitude variations with respect to the position of the virtual source at the reference position are shown in Fig. 2b. Even a quite large radius does not result in significant less artifacts. This is due to the far-field assumptions made in WFS and the spatial sampling of the secondary source contour.

2.5-Dimensional Synthesis

2.5D synthesis aims at producing sound fields within a plane using secondary point sources. Since line sources have a different frequency response and level decay over distance than point sources, mismatches in the synthesized sound field are present [7]. The artifacts of 2.5D synthesis may superimpose to the amplitude deviations caused by an edge in the secondary source contour. Figure 3 shows the relative levels at the reference position for 2D and 2.5D synthesis of a virtual line/point source. The 2.5D driving functions published in [4] have been used. It can be observed that the amplitude variation is increased by about 1 dB in comparison to 2D synthesis.

Case Study – Rectangular Loudspeaker Array

So far, secondary source distributions of semi-infinite length were considered. Practical arrays have a finite number of loudspeakers and are therefore of finite extend. Due to the shape of typical listening rooms, most loudspeaker arrays are of rectangular shape. This also holds for the array at the Institute of Communications Engineering, University of Rostock, consisting of 64-loudspeakers mounted on trusses [8]. Figure 4 depicts the synthesized sound field for 2.5D synthesis of a virtual point source. The irregular sampling has been coped for by weighting the loudspeaker driving signals according to the midpoint rule. The amplitude deviations due to the edge and secondary source type mismatch are clearly visible. In order to further assess the effects at the reference position, the level for various source positions and frequencies has been simulated. Figure 5 shows the results. For frequencies below 1 kHz systematic amplitude variations in the range of ± 3 dB can be observed. Spatial aliasing interfere with these above 1 kHz, resulting in a finer structure in the range of ± 6 dB. Informal listening revealed that the amplitude deviations are clearly audible as loudness changes and coloration.

Conclusions

The foundations of WFS are based on the assumption of a smooth secondary source contour ∂V. The resulting artifacts, when this assumption is violated, have been investigated. Using known driving functions for WFS, the level of the synthesized sound field for a rectangular loudspeaker array deviates considerable from the desired one. These variations depend on the secondary source contour ∂V, the listener position, the type and position of the virtual source, the frequency, the amplitude varia-

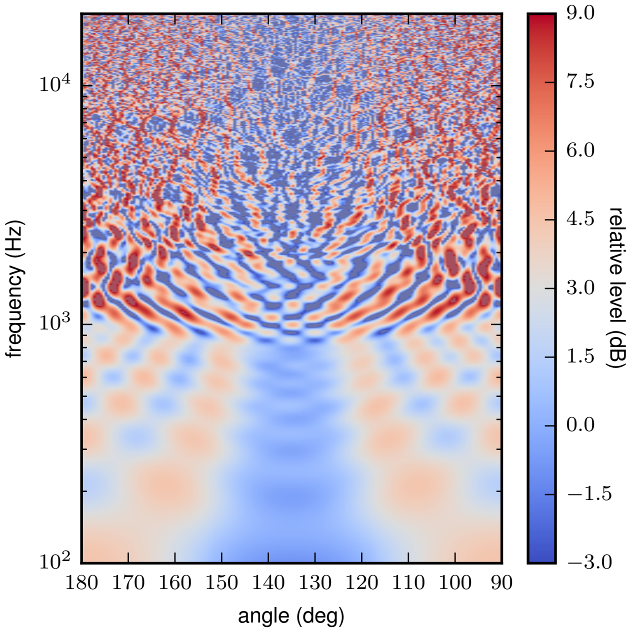


Figure 5: Relative level at the reference position for synthesis of a point source by 2.5D WFS. The source is located at a distance of $R = 4$ m and an angle α from the reference position. The level is normalized to $\alpha = 135°$.

tion due to 2.5-dimensional synthesis and the directivity of the loudspeakers.

The presented investigations raise a number of topics for further research: Is there an optimal spatial sampling scheme of a given secondary source contour and how to determine the discretization weights? Are there improved secondary source selection and tapering schemes for closed non-smooth secondary source distributions? Is it possible to derive alternative and efficient driving functions for non-smooth secondary source distributions? For the latter, first results of the authors using the equivalent scattering approach seem to be quite promising.

Reproducible Research

The numerical simulations base on the Python port of the Sound Field Synthesis Toolbox [9], which is available at [1]. The scripts used for this paper can be downloaded from [2]. Additional resources are available at [3].

Acknowledgments

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[1] https://github.com/sfstoolbox/sfs-python
[2] https://github.com/spatialaudio/non-smooth-secondary-source-distributions
[3] http://spatialaudio.net/non-smooth-secondary-source-distributions/

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